

1. Let A and B be two bounded sets of Real numbers. Suppose

$$A + B := \{a + b : a \in A, b \in B\}.$$

- (i) Show that $\sup(A) + \sup(B) = \sup(A + B)$.
(ii) Suppose $C = \{c \in \mathbb{R} : c = a + b, a^2 < 2, b < 5\}$, find $\sup(C)$.

2. Consider the $\{y_n\}_{n=1}^{\infty}$, such that $y_1 = 1$ and $y_{n+1} := \sqrt{1 + 2y_n}$ for $n \geq 1$. Show that y_n converges and find its limit.

3. Decide (giving adequate justification.) whether the following statements are true or false

(a) Let $\{x_n\}_{n=1}^{\infty}$ be a bounded sequence of real numbers. Let $M = \sup\{x_k : k \geq 1\}$. Then there exists a $\{x_{n_k}\}_{k=1}^{\infty}$ of $\{x_n\}_{n=1}^{\infty}$ such that $x_{n_k} \rightarrow M$ as $k \rightarrow \infty$.

(b) There is no ordering of the field of complex numbers, \mathbb{C} , such that it becomes an ordered field.

(c) Let $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ be two Cauchy sequences, such that $y_n > 0$ for all $n \in \mathbb{N}$. Then $\{\frac{x_n}{y_n}\}_{n \geq 1}$ is always a Cauchy sequence.